Algebra C/D

Holt Public Schools Vision Statement for K-12 Mathematics Instruction:

We believe students in mathematics in Holt Public Schools need a productive disposition towards mathematics and to view themselves as confident mathematicians. In order to build this disposition, students will gain strong conceptual knowledge that then supports development of their procedural skills. Students will make sense of problems and persevere in solving them. In those problems, students will model and reason abstractly and quantitatively. Students will construct viable arguments and critique the reasoning of others.

Math

Tiered Philosophy

In Holt Public Schools, we believe all students are able to become capable mathematicians. We recognize that this does not happen at the same pace for all students, so some students, at various times, will need additional support to be successful. Because we value all students experiencing rigorous math classes with their peers, the support students receive will be in addition to their regular, at-level math course. By increasing the amount of time students engage with mathematics during the day, we are able to help students close existing knowledge gaps that hinder success with their grade level course work, see connections between mathematical ideas, deepen their understanding of current and prior knowledge, and develop a positive mathematical identity.

According to Dr. Rebecca Sarlo, Tier 2 supports and interventions at the secondary level "should be designed to support student success with core instructional content (2014)." The supports should address knowledge or gaps that are more relevant to the current core instruction students are receiving. In addition to supporting students' acquisition of mathematical concepts, students also build their efficacy at being a successful mathematics student. This happens through increasing engagement through goal setting, high quality and high frequency feedback, and students monitoring their own progress.

Students who receive this support at grades 7-9 typically have some gaps in their prior knowledge or underdevelopment of some mathematical habits of mind that will be problematic for future success. Students are identified using data points such as prior course failures, common unit test or exam scores, unit screeners, or teacher recommendation. By utilizing the mathematic support classes, students are engaged in mathematics for more minutes during the day than their peers, which helps to close knowledge gaps. The class sizes are smaller so students receive more frequent teacher feedback. Students engage in the mathematical practice standards and collaborate with their peers in order to become more confident in themselves as capable and successful mathematicians. Teachers organize learning opportunities for students to build their mathematical habits of exploring ideas, orienting/organizing, thinking in reverse, representing, justifying, generalizing, checking for reasonableness, and using mathematical language (Horn 2012). In order to provide these experiences, instruction is not of an "I do, we do, you do" type model.

According to Rollins (2014), support that is remediation of prior content that is not relevant to what the student is expected to do in their current math class only keeps that student behind. She advocates for addressing past conceptual and procedural knowledge gaps connected to the new learning expected students experience in their grade level math class. As a result, the learning opportunities teachers provide are centered on mathematical content that is prerequisite knowledge for what students need to be successful in their core class in real time. This helps students engage in the core instruction with their peers rather than falling further behind and waiting to catch up.

Below are student experiences and related teacher knowledge or actions from literature on best mathematical teaching practices. The resources used to compile this were:

- *Small Steps*, *Big Changes*, Confer and Ramirez (2012)
- *Principles to Actions*, National Council of Teachers of Mathematics (2014)
- *Adding It Up*, National Research Council (2001)
- *Strength in Numbers*, Horn (2012)

We believe all students need to understand the following expectations and engage in these actions at all grades:

Student experiences	Related teacher knowledge or actions
Students justify their mathematical arguments and critique those of others.	• Teachers keep the complexity of authentic learning tasks • Teachers anticipate and use students' errors and misconceptions as learning opportunities • Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify • Teachers have multiple mathematical representations and strategies to help support students in making connections between their mathematical ideas and those of others
Students apply multiple strategies.	• Teachers have a strong understanding of the mathematics they teach and how it connects: concepts, procedures, representations, strategies, language • Teachers gather evidence of knowledge during instruction and use assessment data strategically to help students refine their mathematical knowledge and support building connections between ideas.
Students write, talk about, and present their mathematical ideas.	• Teachers facilitate students making connections between mathematical ideas • Teachers anticipate common mathematical errors and misconceptions, and when students make these, use them as learning opportunities • Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify
Students engage in solving mathematical problems with peers.	• Teachers keep the complexity of authentic learning tasks • Teachers build interdependence among students by facilitating group work and having norms.
Students engage in productive struggle and persevere.	• Teachers have a strong understanding of the mathematics they teach and how it connects (concepts, procedures, representations, strategies, language) in order to facilitate a productive struggle • Teachers keep the complexity of authentic learning tasks to promote productive struggle • Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning in order to know entry points into the problems and suggestions of prior knowledge that

	will help students progress through complex tasks.
Students solve complex problems with multiple solution paths.	• Teachers have a strong understanding of the mathematics they teach and how it connects (concepts, procedures, representations, strategies, language) to allow multiple solution paths • Teachers have multiple mathematical representations and strategies to help teach students • Teachers keep the complexity of authentic learning tasks so there are multiple solution paths • Teachers gather evidence of knowledge during instruction and use assessment data strategically in order to facilitate students seeing a robust set of solution paths
Students create and use visual models and multiple representations.	• Teachers have a strong understanding of the mathematics they teach and how it connects (concepts, procedures, representations, strategies, language) to allow multiple representations • Teachers keep the complexity of authentic learning tasks
Students are self-assessing based on learning goals. Related to students use metacognitive strategies to know when to adjust their learning strategies in relation to learning goals.	• Teachers anticipate common mathematical errors and misconceptions, and when students make these, use them as learning opportunities • Teachers differentiate, when appropriate, for students who are struggling as well as those who need additional challenges
Students value mathematics.	• Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify to provide multiple opportunities for students to see value in multiple aspects of mathematics • Teachers differentiate, when appropriate, for students who are struggling as well as those who need additional challenges
Students believe in their own efficacy.	• Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify to provide multiple opportunities for students to grow their efficacy • Teachers gather evidence of knowledge during instruction and use assessment data strategically in order to provide support to students • Teachers differentiate, when appropriate, for students who are struggling as well as those who need additional challenges • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning in order to support all students at being successful in mathematics
Students will make connections based on **conceptual** understandings.	• Teachers have a strong understanding of the mathematics they teach and how it connects: concepts, procedures, representations, strategies, language • Teachers facilitate students making connections between mathematical ideas • Teachers have multiple mathematical representations and strategies to help teach students • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning

Students make connections between multiple representations.	• Teachers have a strong understanding of the mathematics they teach and how it connects: concepts, procedures, representations, strategies, language • Teachers have multiple mathematical representations and strategies to help teach students • Teachers facilitate students making connections between mathematical ideas in order to connect conceptual understandings to procedural knowledge and connections across mathematical ideas • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning in order to identify the connections students should see

Algebra C/D course overview

Algebra C/D is typically an eleventh grade course, although there is flexibility regarding when the student takes the course. Algebra C/D explores exponential and logarithmic, trigonometric, and parametric function families. Similarly to Algebra A/B, students look at the characteristics of these functions, including patterns in the tables, graph characteristics, and forms of equations, to apply these to writing and solving equations in mathematical and real-world problems. In addition, students explore univariate statistics (including normal distributions) and probability (compound, independent, conditional).

Below is one possible sequence of units. First semester is comprised of trigonometric functions and exponential and logarithmic functions. These units could be taught in any order. Second semester is comprised of parametric functions, univariate statistics and probability. Typically parametric functions come first to build off the functions done first semester; however, occasionally in the past, units second semester have been switched. Probability and statistics can be interchangeable in sequence; typically probability comes first.

Approximate learning timeline

Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun

Exponential and Logarithmic Functions	Trigonometric Functions	Parametric Functions	Probability	Univariate Statistics
Properties of exponential functions' tables, graphs, and equations; apply exponential functions to real-world situations; solve exponential functions relating logarithms as the inverse	Properties of trigonometric functions' tables, graphs, and equations; apply trigonometric functions to real-world situations; solve trigonometric functions; relate trigonometric functions to unit circle	Capstone unit revisiting all previously studied function families and applying them to this new family where an input gives an ordered pair as an output, each determined by their own rule. Properties of this new relationship's tables, graphs, and equations; apply to real-world situations	The likelihood of something happening at random in uncertain contexts as related to independent events and conditional probabilities. Counting principles and expected values are also explored.	Representing data on plots; interpreting the plots. Using statistics (measures of center, measures of spread, shapes of distributions) to describe data. Further exploration with normal distributions as related to percentiles and z-scores.

Algebra CD

***Callie notes in green*. Blue highlighting on "Proficiency level" indicates drafted by Callie and needs proofing/editing.**

Unit: Exponentials and Logarithms

Proficiency level	**CC.9-12.F.BF.5 (+)** Understand the inverse relationships between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).*	HPS assessment question	SAT question along with strand aligned to Neither standard assessed
Advanced	Find all real solutions to any exponential or logarithmic equation with generalized coefficients. (And all of the 3 level.)		
Proficient	Find a solution to exponential and logarithmic equation.		
Developing	Find a solution to exponential OR logarithmic equation.		
Beginning	Find an approximate solution in a table or graph for an exponential equation.	.	

Proficiency level	**CC.9-12.F.IF.8b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay*. **CC.9-12.F.LE.5** Interpret the parameters in a ~~linear or~~ exponential function in terms of a context. **CC.9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.* **CC.9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.*	HPS assessment question	SAT question along with strand aligned to
Advanced	I wanted to make this could use properties of exponents to rewrite a rule with an exponent of (for example) t/10 to have one as just t, but that's a different standard and had been labeled as honors in our alignment doc.		Passport to advanced math

			14 $P = 215(1.005)^{\frac{t}{3}}$ The equation above can be used to model the population, in thousands, of a certain city t years after 2000. According to the model, the population is predicted to increase by 0.5% every n months. What is the value of n ? A. 3 B. 4 C. 12 D. 36
Proficient	Interpret the parameters of an exponential rule with any type of exponent in terms of the context and identify, based on the rule, whether it represents exponential growth or decay.		
Developing	Given an exponential rule with an exponent of a single variable, interpret the parameters of an exponential rule in terms of the context and identify whether it represents growth or decay.		
Beginning	Given an exponential rule with an exponent of a single variable, can interpret the parameters of an exponential rule in terms of the context for growth OR decay OR Given an exponential rule with an exponent of a single variable, can interpret some parameters of an exponential rule in terms of the context for		

	growth and decay OR Given an exponential rule with an exponent of a single variable, can interpret as growth or decay		

Proficiency level	**CC.9-12.F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. **CC.9-12.F.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **CC.9-12.F.LE.1a** Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. How would we expect kids to "prove" this? I would anticipate the results of this coming up in conversation though, so I put it here. **CC.9-12.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially	HPS assessment question	SAT question along with strand aligned to

	eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		
Advanced	Prove that an increasing exponential function eventually exceeds any increasing polynomial function.		
Proficient	Identify a table, graph, or situation as linear or exponential. If exponential, identify whether it is growth or decay, and find the constant percent rate/constant multiplier/what vocab term you use with kids		Problem Solving and Data Analysis **25** Which of the following describes an exponential relationship between the pair of variables listed? A. For every 3-millimeter increase *m* in the thickness of a piece of glass, the intensity of light *I* traveling through the glass decreases by 20%. B. Each second *s*, a car's speed *C* decreases at a constant rate of 10 meters per second. C. With every 33-foot increase in depth *d* below the surface of water, the pressure *p* on an object increases by 14.7 pounds per square inch. D. The depth *d* of water remaining in a reservoir decreases by 15 inches each minute *m* as the water is being pumped out at a constant rate. **15** $\begin{array}{c\|c\|c\|c\|c\|c} x & 1 & 2 & 3 & 4 & 5 \\ \hline y & \frac{11}{4} & \frac{25}{4} & \frac{39}{4} & \frac{53}{4} & \frac{67}{4} \end{array}$ Which of the following equations relates *y* to *x* for the values in the table above? A. $y=\frac{1}{2}\cdot\left(\frac{5}{2}\right)^x$ B. $y=2\cdot\left(\frac{3}{4}\right)^x$ C. $y=\frac{3}{4}x+2$ D. $y=\frac{7}{2}x-\frac{3}{4}$
Developing	Identify a table, graph, or situation as linear or exponential. If		

	exponential, identify whether it is growth or decay.		
Beginning	Identify a table, graph, or situation as linear or exponential.		

These all felt similar to me in terms of looking at function representations and determining whether linear or exponential and that exponential grows quickest of all the function families they've seen.

Proficiency level	**CC.9-12.F.LE.2** Construct ~~linear and~~ exponential functions, ~~including arithmetic and geometric sequences~~, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.*	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Write any exponential rule from any table, graph, or situation. Use it to evaluate for a given input.		
Developing	Write any exponential rule from some tables, graphs, or situations. OR Write growth or decay exponential rules from any table, graph, or situation. AND Use it to evaluate for a given input.		
Beginning	Identify the value to put in for *b* in the equation when given the constant percent rate/constant multiplier/what vocab term you use with kids		

I wondered about the varying complexity of the representations they're given being the distinguishing parts between the levels…?

Proficiency level	**CC.9-12.F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).*	HPS assessment question	SAT question along with strand aligned to F.LE.4 not assessed
Advanced	Find all solutions to any exponential equation with generalized coefficients OR write an inverse for an exponential rule with a complex exponent (expression rather than just a variable), and use it to solve exponential equations.		
Proficient	Write an inverse equation for an exponential rule; use it to solve exponential equations.		
Developing	Write an inverse equation for an exponential rule.		
Beginning	Find an approximation solution to an exponential equation in a table or graph.		

Proficiency level	Standard: **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~	HPS assessment question	SAT question along with strand aligned to
Advanced	Based on given function representation, justify all of the characteristics below OR can describe all the possibilities for a general rule.		
Proficient	Based on given function representation, explain all of the following for exponential and logarithmic: coordinates of the x- and y-intercepts when they exist, end behavior, and domain and range. Graph (sketch or on axis) the rule.		
Developing	Based on given function representation, determine most of the following: coordinates of the x- and y-intercepts when they exist, end behavior, and domain and range. Graph (sketch or on axis) the rule.		
Beginning	Based on given function representation, determine any of the following: coordinates of the x- and y-		

	intercepts when they exist, end behavior, and domain and range. Graph (sketch or on axis) the rule.		

Explain: "It has an asymptote on the x-axis because the y-values can't ever hit 0," is a 3 because it gets at why the kid gave that answer.
Justify: "It has an asymptote on the x-axis because the y-values can't ever hit 0 because negative exponents work like division so you cut the value of the y-intercept into more pieces but you can't ever cut it enough times to where the pieces are size 0," is a 4 because it delves further into the mathematical reason.

Check this for agreement on difference between explain and justify.

Unit: Circular Trigonometry

<table>
<tr><td>Proficiency level</td><td>Standard: F.IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

CC.9-12.F.IF.7e

Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.</td><td>HPS assessment questions</td><td>SAT question along with strand aligned to</td></tr>
<tr><td>Advanced</td><td>Based on given function representation, justify all of the characteristics below OR can describe all the possibilities for a general rule.</td><td></td><td></td></tr>
<tr><td>Proficient</td><td>Based on given function representation, explain all of the following: period, amplitude, midline, and phase shift. Graph (sketch or on axis) the rule.</td><td rowspan="3">What is the maximum of the function ?

What is the minimum of the function ?

What is the period of ?

Given and which of the following would be different for these functions?</td><td></td></tr>
<tr><td>Developing</td><td>Based on given function representation, determine most of the following: period, amplitude, midline, and phase shift. Graph (sketch or on axis) the rule.</td><td></td></tr>
<tr><td>Beginning</td><td>Based on given function representation, determine any of the following: period,</td><td></td></tr>
</table>

	amplitude, midline, and phase shift. Graph (sketch or on axis) the rule.	What is the amplitude of the function represented in the graph below? What is the period of the function represented in the graph below?	

Proficiency level	Standard**: CC.9-12.F.TF.7** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* **CC.9-12.F.TF.6** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).*	HPS assessment questions	SAT question along with strand aligned to F.TF.7 and F.TF.6 are both + standards so not assessed on SAT. F.BF.4a is not assessed on SAT.
Advanced	Find all solutions to any trigonometric equation or find all solutions in a given domain.		
Proficient	Find two solutions in a single period to a trigonometric equation in any form.		
Developing	Find a solution in a single period to a trigonometric equation in any form.	Find a solution, rounded to the nearest thousandth, to . Find a solution, rounded to the nearest thousandth, to 15 Solve:	

Beginning	Find an approximate solution in a table or graph for a trigonometric equation.	For any multiple choice questions, since we can’t see how students are solving, it’s possible the level 2 questions are assessing this level 1.	

Proficiency level	Standard: **CC.9-12.F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.*	HPS assessment questions	SAT question along with strand aligned to F.TF.5 is not assessed on the SAT. Students do F.BF.1 in all units.
Advanced			
Proficient	Given any trigonometric function representation (table, graph, situation), write an algebraic rule.	Write a rule for the following function:	
Developing	Write a rule from any 2 of the 3 representations.		
Beginning	Write a rule from any 1 of the representations.		

There was a conversation about phase shift since it's not listed in the standard. Are we still going to do this? Do we have common expectations for kids regarding this?

Proficiency level	Standard: **CC.9-12.F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **CC.9-12.F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **CC.9-12.F.TF.4** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	HPS assessment questions	SAT question along with strand aligned to F.TF.4 is a + standard so not assessed on SAT.
Advanced			
Proficient	Calculate values of trigonometric functions at locations on a circle that correspond to multiples of[Symbol]/2 relating this to a unit circle (or similar circle).	Solve:	
Developing	Calculate the location on a circle in radians given the degree rotation and vice versa. Relate the unit circle to trig functions in real numbers.	Find the radian measure of Find the degree measure of	Q19 and Q20 were with no calculator.

Beginning	Explain how a radian measure is related to an amount of degree rotation around a circle. Identify locations of radian measures on a unit circle.	Which circle below shows the approximate location of a point that has rotated radians from Standard Position? Which circle below shows the approximate location of a point that has rotated radian from Standard Position?	

Unit: Parametrics

Students revisit a lot of their knowledge of linear, quadratic, and rational functions. Students and teachers would reuse some of those learning progressions. (We should probably identify which. Do we want to recopy them here?) Writing the x(t) and y(t) rules would just be writing a rule from a family of function they already studied, right? Even evaluating and solving them would be old learning progressions.

Proficiency level	Standard: **CC.9-12.F.BF.1c** (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	HPS assessment question	SAT assessment question and strand aligned to Since it is a + standard, this is not assessed on SAT.
Advanced			
Proficient	Given an $x(t)$ and a $y(t)$ rule, write a $y(x)$ rule.		
Developing			
Beginning			

Unit: Probability

Proficiency level	Standard: **CC.9-12.S.MD.3** (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.** **CC.9-12.S.MD.4** (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?** **CC.9-12.S.MD.2** (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.* **CC.9-12.S.MD.5** (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.*	HPS assessment question	SAT assessment question and strand aligned to All are + standards so SAT does not assess

	CC.9-12.S.MD.5a (+) Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **CC.9-12.S.MD.5b** (+) Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*		
Advanced	Given a specific expected value, determine a value for a specific event or probability.		
Proficient	Develop the probability distribution (the different possible events and assign their given probabilities) and find an expected value. Interpret or describe what the expected value says about the event.		
Developing	Develop the probability distribution and calculate the expected value.		
Beginning	Develop the probability distribution.		

Proficiency level	Standard:	HPS assessment question	SAT assessment question and strand aligned to
	CC.9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **CC.9-12.S.CP.3** Understand the conditional probability of A given B as P(A and B)/P(B), ~~and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.~~ **CC.9-12.S.CP.5** Recognize and explain the concepts of conditional probability ~~and independence~~ in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung*		S.CP. 3 and S.CP.5 and S.CP.6 are not assessed

<table>
<tr><td></td><td>cancer.

CC.9-12.S.CP.6

Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.*</td><td></td><td></td></tr>
<tr><td>Advanced</td><td>Given a conditional probability and other sufficient information, work backwards to find a different probability from the sample space. (Tweak still)</td><td></td><td></td></tr>
<tr><td>Proficient</td><td>Find the conditional probability from any representation. Differentiate between conditional probabilities in a context: P(A|B) versus P(B|A).</td><td></td><td></td></tr>
<tr><td>Developing</td><td>Find the conditional probability in the wrong events: example, when asked for P(A|B), they give P(B|A).</td><td></td><td></td></tr>
<tr><td>Beginning</td><td>Give a probability out of the entire sample space rather than the subset for conditional.</td><td></td><td>Problem solving and data

13
<table>
<tr><td></td><td colspan="3">Roof type</td><td></td></tr>
<tr><td></td><td>Asphalt shingle</td><td>Slate</td><td>Cedar shake</td><td>Total</td></tr>
<tr><td>Single story</td><td>9</td><td>4</td><td>2</td><td>15</td></tr>
<tr><td>Two story</td><td>20</td><td>10</td><td>3</td><td>33</td></tr>
<tr><td>Total</td><td>29</td><td>14</td><td>5</td><td>48</td></tr>
</table>
The table above shows the distribution of single-story and two-story houses in a neighborhood classified according to roof type. If one of the houses is selected at random, what is the probability that it will be a single-story house with a slate roof?

A. $\frac{4}{48}$
B. $\frac{4}{15}$
C. $\frac{4}{14}$
D. $\frac{14}{48}$</td></tr>
</table>

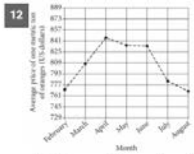
Proficiency level	Standard:	HPS assessment question	SAT assessment question and strand aligned to
	CC.9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **CC.9-12.S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. However, HHS math dept chooses to do this one: P(A|B)=P(A) because this allows students to continue to revisit conditional probability and reinforces the definition of what it means for things to be independent – that A still happens with the same likelihood even if B happened already. **CC.9-12.S.CP.3** ~~Understand the conditional probability of A given B as P(A and B)/P(B),~~ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the		S.CP.2, S.CP.3, S.CP.5, and S.CP.8 are not assessed

	probability of B. **CC.9-12.S.CP.5** Recognize and explain the concepts of ~~conditional probability and~~ independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **CC.9-12.S.CP.8** (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)], and interpret the answer in terms of the model.		
Advanced	Ask a probability independence question, mathematize it, and come to a conclusion about independence. (Ex: does the cold affect Brett Favre's completion rate?)		
Proficient	Compares appropriate probabilities to determine whether two events are independent and interpret independence in a context.		
Developing	Compares probabilities inappropriately and uses this comparison as justification for independence.		
Beginning	No mathematical justification for independence, relates more to cause and effect or intuition.		

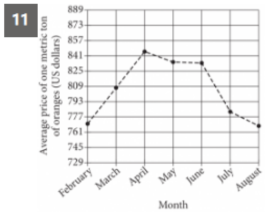
Proficiency level	Standard: **CC.9-12.S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). **CC.9-12.S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **CC.9-12.S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.* **Students are not limited to 2-way tables to show sample spaces. They are expected to be able to represent situations in any representation.	HPS assessment question	SAT assessment question and strand aligned to S.CP.1 and S.CP.7 are not assessed
Advanced	Find a probability that requires working solving from other probabilities.		
Proficient	Create a sample space and find the probabilities of desired outcomes (unions, intersections, complements).		
Developing	Can find P(A) but has errors when computing P(A and B) or P(A or B).		
Beginning	Uses intuition or assumes all events equally likely to create sample spaces. Calculates probability based on that.		

Unit: Statistics

Proficiency level	Standard: **CC.9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots). **CC.9-12.S.ID.3** Interpret differences in shape, ~~center, and spread in the context of the data sets,~~ accounting for possible effects of extreme data points (outliers).	HPS assessment question	SAT assessment question and strand aligned to
Advanced	Describe advantages and disadvantages of data plots in relation to the information you can glean from the representation.		
Proficient	Create a data plot and describe the shape and possible effect of extreme data points.		
Developing	Create data plot accurately but describe with non-standard or inaccurate terminology.		
Beginning	Plot without proper scale on axes. OR Read variety of plots to identify information.		

Proficiency level	Standard: **CC.9-12.S.ID.3** Interpret differences in ~~shape~~, center, ~~and spread~~ in the context of the data sets, accounting for possible effects of extreme data points (outliers).	HPS assessment question	SAT assessment question and strand aligned to
Advanced	Understand how a transformation on the data affect the measures of center (adding 5 to all the data, increasing all the data by 10%, ignoring an extreme value).		Problem solving and data 12  The line graph above shows the average price of one metric ton of oranges, in dollars, for each of seven months in 2014. In 2014, the average price of one metric ton of oranges decreased by 2.36% from January (not shown) to February. Which of the following is closest to the price of one metric ton of oranges in January 2014? A. 700 B. 770 C. 790 D. 830
Proficient	Calculate and distinguish between the mean, median, and mode. Explain what the measures of center represent. Explain the effect of extreme values on the measures of center.		
Developing	Calculate measures of center accurately.		Problem solving and data

11



The line graph above shows the average price of one metric ton of oranges, in dollars, for each of seven months in 2014.

Which of the following is closest to the median price, in dollars, of the seven recorded prices of one metric ton of oranges?

A. 834

B. 808

C. 783

D. 768

6

Nutritional Information for 1-Ounce Servings of Seeds and Nuts

Seed or nut	Calories	Total fat (grams)	Protein (grams)
Pecan	198	20.2	3.0
Pistachio	80	6.5	3.0
Pumpkin	159	13.9	8.5
Sunflower	166	14.6	5.9
Walnut	185	18.5	4.3

The table above shows the calories, grams of fat, and grams of protein in 1-ounce servings of selected seeds and nuts.

Lionel purchases 1-pound bags of each of the five seeds and nuts shown in the table. Of the following, which best approximates the average (arithmetic mean) number of calories per bag? (1 pound = 16 ounces)

A. 150

B. 250

C. 1,500

D. 2,500

35

Ticket Prices by Row Number

Row number	Ticket price
1–2	$25
3–10	$20
11–20	$15

The price of a ticket to a play is based on the row the seat is in, as shown in the table above. A group wants to purchase 10 tickets for the play.

They will purchase 3 tickets for seats in row 1.

They will purchase 2 tickets for seats in row 3.

They will purchase 2 tickets for seats in row 4.

They will purchase 3 tickets for seats in row 12.

What is the average (arithmetic mean) ticket price, in dollars, for the 10 tickets? (Disregard the $ sign when gridding your answer.)

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Number of States with 10 or More Electoral Votes in 2008

Electoral votes	Frequency
10	4
11	4
12	1
13	1
15	3
17	1
20	1
21	2
27	1
31	1
34	1
55	1

In 2008, there were 21 states with 10 or more electoral votes, as shown in the table above. Based on the table, what was the median number of electoral votes for the 21 states?

A. 13

B. 15

C. 17

D. 20

36

	Masses (kilograms)					
Andrew	2.4	2.5	3.6	3.1	2.5	2.7
Maria	x	3.1	2.7	2.9	3.3	2.8

Andrew and Maria each collected six rocks, and the masses of the rocks are shown in the table above. The mean of the masses of the rocks Maria collected is 0.1 kilogram greater than the mean of the masses of the rocks Andrew collected. What is the value of x ?

Beginning	Incomplete conceptual understanding of center (ex.		

	Find the median wrong, not know what to do when an even amount of data points)		

Proficiency level	Standard: **CC.9-12.S.ID.3** Interpret differences in ~~shape, center, and~~ spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	HPS assessment question	SAT assessment question and strand aligned to
Advanced	Understand how a transformation on the data affect the measures of spread (adding 5 to all the data, increasing all the data by 10%, ignoring an extreme value).		
Proficient	Calculate and distinguish between standard deviation, range, and IQR. Explain what the measures of spread represent. Explain the effect of extreme values on the measures of spread.		Problem solving and data 17 Data set A25,55040,430 49,150 62,590 73,670 118,780126,040 Data set B22,86055,020173,730300,580358,920456,170603,300 Which of the following is true about the standard deviations of the two data sets in the table above? A. The standard deviation of data set B is larger than the standard deviation of data set A. B. The standard deviation of data set A is larger than the standard deviation of data set B. C. The standard deviation of data set A is equal to the standard deviation of data set B. D. There is not enough information available to compare the standard deviations of the two data sets.
Developing	Calculate measures of spread accurately.		
Beginning	Find range and distinguish it from the range of a function		

Proficiency level	Standard: **CC.9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **This gets at the writing about statistics. Possibly add	HPS assessment question	SAT assessment question and strand aligned to

	in the math practice about justifying?		
Advanced	Create a question to research and begin to answer with statistical analysis.		
Proficient	Uses statistics beyond mean and median to make statements comparing two data sets. Values are cited accompanied by an explanation about what they mean about the data set and situation. Outliers are addressed.		
Developing	Speaks to measures of spread and center but only compares values to each other without elaboration about what the values represent about the context.		
Beginning	Uses mean, median, max, or min to compare data sets.		

Proficiency level	Standard: **CC.9-12.S.ID.3** ~~Interpret differences in shape, center, and spread in the context of the data sets,~~ accounting for possible effects of extreme data points (outliers)	HPS assessment question	SAT assessment question and strand aligned to
Advanced	Can create a data set where values are exist as outliers under one test but not the other.		
Proficient	Can identify outliers in a given set of data based on both the 3-sigma and 1.5IQR tests.		
Developing	Calculate the cutoff points where outliers may begin without identifying specific points from the data; or, misconceptions like adding the 1.5IQR to the median.		
Beginning	Identify values that intuitively appear to be extreme.		

Proficiency level	Standard: **CC.9-12.S.ID.4** Use the mean and standard deviation of a data set ~~to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve~~ Calculate and interpret Z-scores	HPS assessment question	SAT assessment question and strand aligned to Not assessed
Advanced	Given a z-score and additional piece of info about the data set, work backwards to find the other.		
Proficient	Recognize that every data point has a z-score; Calculate z-scores and explain what it tells you about that data value in context.		
Developing	Calculate z-scores for data values.		
Beginning	Find deviations for data values.		

Proficiency level	Standard: **CC.9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution ~~and to estimate population percentages~~. Recognize that there are data sets for which such a procedure is not appropriate. ~~Use calculators, spreadsheets, and tables to estimate areas under the normal curve.~~	HPS assessment question	SAT assessment question and strand aligned to Not assessed
Advanced	Quantitatively analyze a set of data to determine if it is approximately normal.		
Proficient	Understand characteristics of data in order to be approximately normal (bell shape that follows the "Empirical Rule").		
Developing	Identifying symmetrical distributions or a bell shape as normal.		
Beginning	Recognize that there are a variety of distribution shapes.		

Proficiency level	Standard: **CC.9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	HPS assessment question	SAT assessment question and strand aligned to Not assessed
Advanced	Given percentages can find data values or statistic for a normal data set.		
Proficient	Confirms approximately normal; finds percentile and corresponding percentages for one or more data values (ex: what percent of the data lies between…)		
Developing	Find a z-score for a data value and find associated percentile.		
Beginning	Using data at 1, 2, and 3 standard deviations to give percentages of data within different intervals defined by these values.		

Proficiency level	Standard: **CC.9-12.S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	HPS assessment question	SAT assessment question and strand aligned to
Advanced			
Proficient			Problem solving and data **4** One hundred park-district members will be selected to participate in a survey about selecting a new park-district coordinator. Which of the following methods of choosing the 100 members would result in a random sample of members of the park district? A. Obtain a numbered list of all park-district members. Use a random number generator to select 100 members from the list. Give the survey to those 100 members. B. Obtain a list of all park-district members sorted alphabetically. Give the survey to the first 100 members on the list. C. Tell all park-district members that volunteers are needed to take the survey. Give the survey to the first 100 members who volunteer. D. Obtain a list of all park-district members who are attending an upcoming event. Give the survey to the first 100 members on the list. **10** To determine whether residents of a community would vote in favor of a ballot proposal to use \$100,000 of local taxes for additional playground equipment at a community park, Jennifer surveyed 60 adults visiting the park with their children during one week in June. She found that 45 of those surveyed reported that they would vote in favor of the proposal. Which of the following statements must be true? A. When the actual vote is taken, 75 percent of the votes will be in favor of the proposal. B. No prediction should be made about the vote on the proposal because the sample size is too small. C. The sampling method is flawed and may produce biased results. D. The sampling method is not flawed and is likely to produce unbiased results.
Developing			
Beginning			

Parent Resources:

- ?